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Geometric Portfolio Optimization based on Skewness Maximization and Monte Carlo Simulation

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Abstract

Optimum portfolio selection is one of the financial issues that has been studied in various ways in recent decades. One of the important factors in the portfolio selection is to consider the behavior of returns distribution. In this paper, it is assumed that returns are skew normal distribution, and under skewness with risk optimum portfolio is examined. Optimality and simulation results considering the skewness showed that this method is more effective than Markowitz traditional method.

Keywords: Portfolio optimization, Geometric mean return, Monte Carlo simulation - Skew-normal distribution.

1. Introduction

Nowadays investors are interested in portfolio optimization in order to increase their wealth and payout. On the other hand, selecting portfolio with various stocks results in risk reduction under some circumstances. Modern portfolio theory first presented by Markowitz in the mid-19th century, made a great evolution in solving portfolio optimization issues. Markowitz model evaluated two main concepts simultaneously: risk (variance) and stock returns mean (average). Markowitz model works under good conditions. They include: Risk aversion, Normal distribution of stock returns, efficient market, low correlation between stock returns, and etc. However, it may not occur in reality. For example if return distribution isn't normal or it has skewness, the intended aim won't happen. Some years later, pricing theory presentation improved Markowitz model, so other conditions were presented to solve the problem. In this research there is a new idea in which return distribution is assumed to have skew normal distribution and skewness is used as an evaluation criterion and Enhanced ability to model. Skew- normal distribution was first presented by Azzalini in 1984 [1] in order to modeling the behavior of variables that have an asymmetric structure. Various efforts have been done on optimizing portfolio context. Xiaoxia Huang [18] presented a new definition of risk based on the variance, semi-variance and the possibilities of new models in solving this problem, and also used the hybrid model. Walter J. Gutjahr and et al. [15] solved this problem by using multi-criteria decision model, and with the benefit of Pareto distribution as an economic distribution. Yong Hyun Shin and his colleagues [16] involved in solving optimization problem due to the retirement period daily consumption, and also paying attention to the relative risk aversion. Jun Li and his colleagues [6] have solved optimization problem by using multi-criteria solution, and benefit from the phase of randomization returns and using genetic algorithms.

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Silvia Dedu and et al. [11] have proposed a new model to solve optimization problem by using multiple criteria decision making and conditional value at risk, and also integrating them together. There were several methods to solve optimization problems in recent years that many of these were based on numerical solution, using genetic algorithms, and applying fuzzy method. For example Bhattacharya and his colleagues used mutual entropy and fuzzy triangular numbers as stock returns, and also used genetic algorithm of stock prices in Mumbai to solve optimization problem. Since the returns are generally asymmetric distribution, skewness coefficient was added to the model which has been received more satisfactory results than the traditional model. Using coefficient of skewness as important factor in solving optimization problem is considered. For example, there was a research by Chang and his colleagues in which they used systematic skewness and numerical methods, then they found there was superiority in applying skewness for traditional model correction. Model introduction and related concepts with skew-normal distribution in solving portfolio optimization problem will be discussed in section 2. Results of simulation and also efficient frontier diagrams will be shown in section 3. Further suggestions will be proposed in section 4 as a final section.

2. Model

Consider an investor will design to have an investment in the defined time interval period. Suppose X is a portfolio vector, and μ is the random return mean vector on probability space with cumulative distribution . So final returns would be

$$X^T\mu \tag{1}$$

$$S(x) = E(\log(x^T\mu)) \tag{2}$$

$E(.)$ shows expectation operator. One of the criteria for solving portfolio optimization problem which has been considered by Markowitz proposed method is labor markets and normal stock returns condition. Whatever selective share correlation is less, composed portfolio works best. But in fact, there are some examples in which we are dealing with low symmetric distribution, skew or skew- symmetric. For example, when return distribution is normal but it has skewness. Skewness study can be very useful as simultaneous evaluation criterion for location and scale. So, it is assumed that return distribution is skew. The random variable will have skew-normal distribution if its density function is as follows

$$f(x|\lambda) = 2\phi(x)\Phi(\lambda x) \quad -\infty < x < \infty \tag{3}$$

Where ϕ and Φ denote the density and the cumulative distribution function of standard normal distribution, respectively. If the random variable Z has skew-normal distribution with skewness parameter λ, it will be shown:

$$X \sim SN(\lambda) \tag{4}$$

According to the above definition and based on Markowitz portfolio optimization original model, we are trying to make a model in which if stock returns distribution has skewness, portfolio optimization problem will have more desirable result. So, we will simulate random samples from skew-normal distribution.

$Z \sim SN(\lambda)$ and $\lambda = \lambda(\delta) = \frac{\delta}{\sqrt{1-\delta^2}}$. By these assumptions for portfolio optimization, proposed model is considered as follows:

$$min var(X) \tag{5}$$

$$max skew(X) \tag{6}$$

$$S(X) > \gamma \tag{7}$$

This is equivalent to the following problem:

$$min(var(X) - skew(X)) \tag{8}$$

$$S(X) > \gamma \tag{9}$$

Where γ is a fixed value to represent minimum amount of expectation. Based on Lagrange method Optimization, a portfolio that applies the following criteria is considered as the optimal portfolio:

$$\frac{\partial E(X^T\mu - E(X^T\mu))^2 - E(x^T\mu - E(X^T\mu))^3 + \alpha_1 E(\log(X^T\mu) - \gamma) + \alpha_2(\sum_{i=1}^{10} x_i - 1) + \alpha_3 \sum_{i=1}^{10} x_i}{\partial X} \tag{10}$$

$$= 2E(X^T\mu - E(X^T\mu)) - 3E(X^T\mu - E(X^T\mu))^2 + \alpha_1 E(\frac{\mu}{X^T\mu}) + \sum_{i=1}^{10}(\alpha_2 + \alpha_3)e_i = 0$$

Where $e_i = (0, \ldots, 1, \ldots, 0)^T, i = 1, \ldots, 10$.

Monte Carlo simulation is used extensively in mathematical finance and economics. According to above assumptions we chose different stocks for a portfolio of ten shares to returns with skew-normal distribution, and generate 100,150,200 random samples. MATLAB software is used for simulation and Lingo software also is used for solving the below optimization problem.

$$min\frac{1}{n}(\sum_{i=1}^{10} x_i^2\sigma_i^2 - x_i^3\sigma_i^3) \tag{11}$$

$$\frac{1}{n}\sum_{j=1}^{N}\sum i = 1^{10}\log(x_i^T\mu_i) > \gamma \tag{12}$$

$$\sum_{i=1}^{10} x_i = 1 \tag{13}$$

$$x_i \geqslant 0, \ \ i = 1, \ldots, 10 \tag{14}$$

Its results were shown in following section.

3. Monte Carlo Simulation results

The random samples generate from skew-normal distribution with 100,150,200 pieces samples (There is a correlation matrix in the appendix). Comparative results between Markowitz mean variance model and this research model, in which with the behavior of stock returns have been benefited from skewness criteria in optimization model, shows that this research model has more desirable result than Markowitz model considering skewness criteria in shares distribution. In order to generalize these conditions, efficient frontier is designed for all different conditions in this optimization problem that shows relative efficiency in this model clearly. Coefficients of each stock (portfolio) will be acquired for any amounts of return and expected risk by using the efficient frontier.

Results of simulation for 100 independent random sample:

Table 1: results of simulation for 100 random samples

N	Model	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	Portfolio optimal value
100	Markowitz model	0.074	0.066	0.094	0.066	0.201	0.108	0.087	0	0.019	0.282	0.068
100	This research model model	0.052	0.068	0.071	0.05	0.251	0.1	0.062	0	0	0.34	0.058

Results of simulation for 150 independent random samples:

Results of simulation for 200 independent random samples:

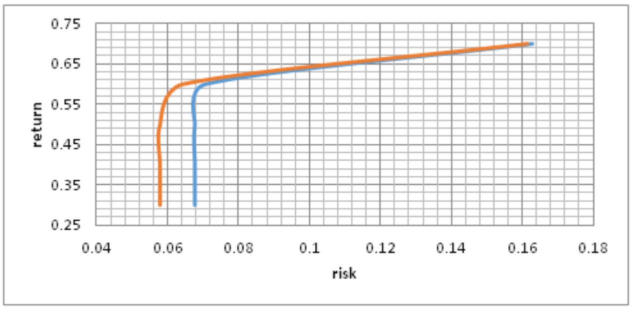

Figure 1: comparing graphs: the red graph presents this research model and the blue graph presents the Markowitz model – sample size=100

Table 2: results of simulation for 150 random samples

N	Model	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	Portfolio optimal value
150	Markowitz model	0.0904	0.097	0.122	0.14	0	0.132	0.15	0.053	0.108	0.106	0.073
150	This research model model	0.0892	0.097	0127	0.14	0	0.1277	0.144	0.051	0.109	0.113	0.072

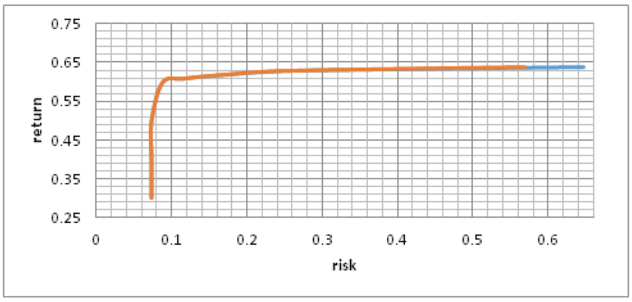

Figure 2: comparing graphs: the red graph presents this research model and the blue graph presents the Markowitz model – sample size=150

Table 3: results of simulation for 200 random samples

N	Model	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	Portfolio optimal value
200	Markowitz model	0.14	0.095	0.121	0.084	0	0.103	0.123	133	0.104	0.094	0.0705
200	This research model model	0.14	0.096	0.113	0.087	0	0.105	0.122	0.137	0.1	0.094	0.0683

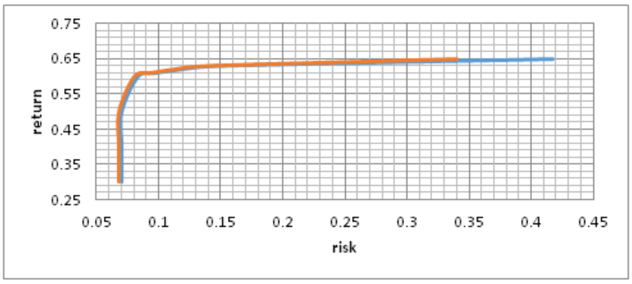

Figure 3: comparing graphs: the red graph presents this research model and the blue graph presents the Markowitz model – sample size=200

4. Further suggestion and conclusion

It was investigated how to use the stock returns and effective factors in final model in order to select the optimum portfolio in this research. According to above tables and diagrams, it was clear that skewness in returns distribution is an important factor. If it is used in final model, it can lead to better stock portfolio selection. On the other hand, it has less variance and risk compared to Markowitz mean-variance model. Pattern of behaviors of each of related criteria with stock selection, such as returns, risk, desirability level, and other qualitative factors can be effective in stock selection process, and also can reduce risk-averse investor's risk.

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